Automatic Control

A brief introduction

Control & time... a long history

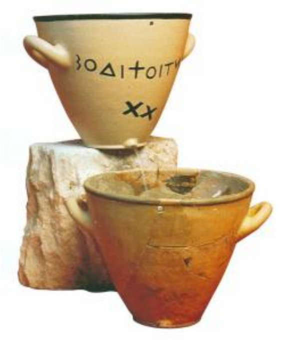


Figure: Water clock

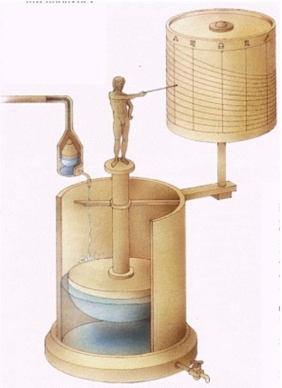


Figure: Clepsydra of Ktesibios(-270 B.C., Alexandria)

Analog closed loop

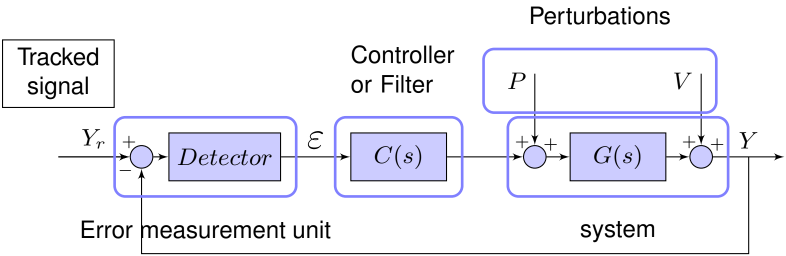


Figure: Main parts.

Controller specifications

In closed loop :

$$C_L(s) = \frac{C(s)G(s)}{1 + C(s)G(s)}$$

$C(s)$ is then choose to fit :

- Stability
- Temporal specifications (rise time, settling time, overshoot...)
- Frequency specifications (cutoff frequency, attenuation...)
- Noise specifications (Equivalent Noise Bandwith...)

Temporal specifications

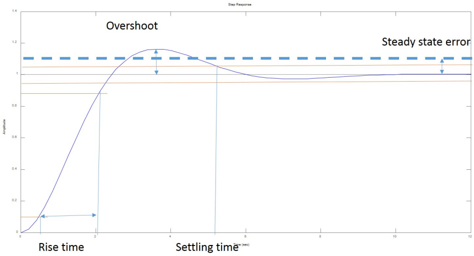


Figure: Time response.

- Overshoot $\simeq 5\%$
  - Settling time $\searrow$
- Settling time : as short as possible
  - Rejection of perturbations
  - more noise !!!
- Steady state error =0

Closed Loop : $C_L \simeq \frac{1}{1+\frac{2\xi}{\omega_0}s+\frac{s^2}{\omega_0^2}}$

Frequency specifications

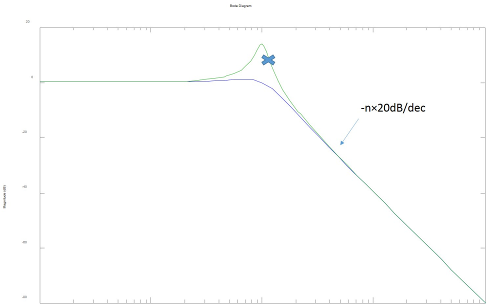


Figure: Frequency response.

- Resonance $\simeq 0\%$
- Bandwith $\searrow$
  - Less noise
- Bandwith $\nearrow$
  - More perturbation rejection
  - Less settling time
  - Decrease lock time (PLL)

Closed Loop : $C_L \simeq \frac{1}{1+\frac{2\xi}{\omega_0}s+\frac{s^2}{\omega_0^2}}$

Equivalent Noise Bandwith

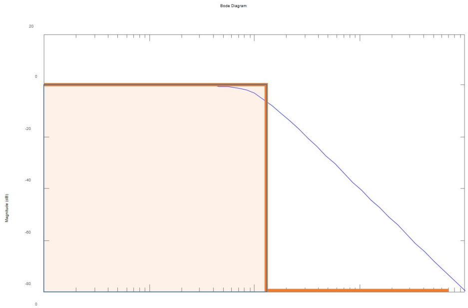


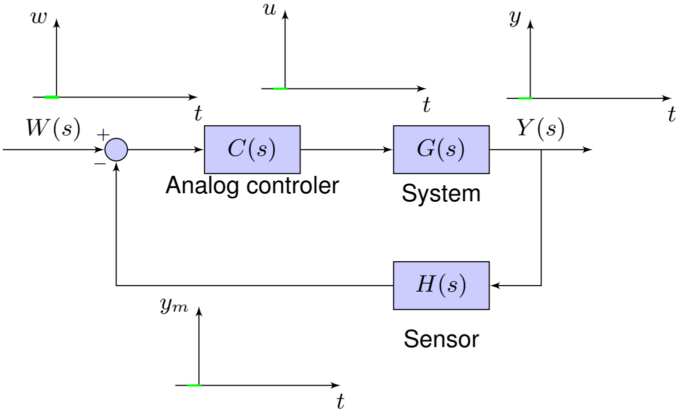
Figure: Equivalent Noise Bandwith.

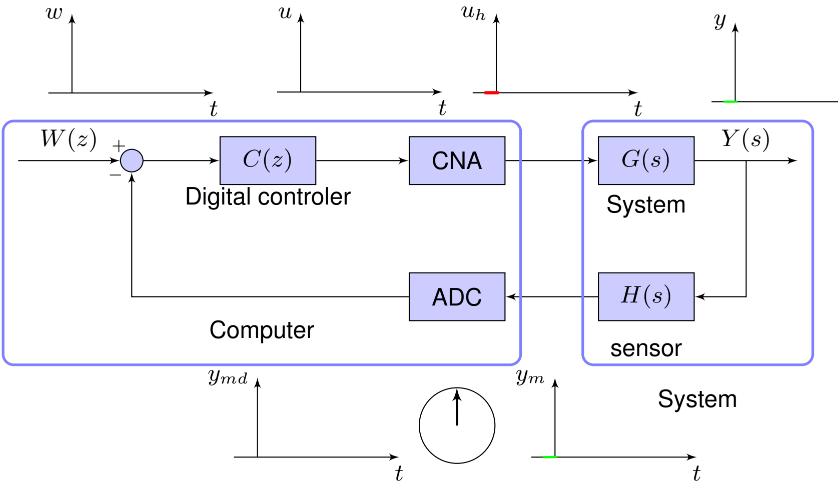
- First order
  $C_L = \frac{1}{1+\tau s}$
  $B_N = \frac{\pi}{2} f_{-3dB} = \frac{1}{4\tau}$
- Second Order
  $C_L = \frac{\frac{2\xi}{\omega_0}s+1}{1+\frac{2\xi}{\omega_0}s+\frac{s^2}{\omega_0^2}}$
  $B_N = \frac{\omega_0}{2}\left(\xi + \frac{1}{4\xi}\right)$

Controller specifications

Closed Loop : $C_L \simeq \frac{1}{1+\frac{2\xi}{\omega_0}s+\frac{s^2}{\omega_0^2}}$

- Stability
- Temporal specifications
  - $\xi = 0.707$ optimal in terms of settling time
- Frequency specifications
  - No resonance $\Longrightarrow \xi > 0.707$
- Noise specifications
  - $\xi = 0.5$ is optimal


w
t
u
t
y
t
W(s)
+
−
C(s)
Analog controler
G(s)
System
Y(s)
H(s)
Sensor
y_m
t


w
t
u
t
u_h
t
y
$W(z)$
+
−
$C(z)$
CNA
Digital controler
$G(s)$
$Y(s)$
System
$H(s)$
ADC
Computer
sensor
y_{md}
t
y_m
t
System

Pros & cons

- Pros
  - New methods of control
  - No parameters variation in the regulator
  - Easy to deal with non-linearities, delays...
  - Multivariable
  - Easy to change the controller
- Cons
  - Discontinuous observation of the output
  - System in open loop between two samples
  - Non-linearities (converters)
  - Signal quantification.
  - Spectral limitation
  - Complex for simple correctors
  - Quantification noise

Controllers technology

Figure: Dspace - NI - Matlab.

- powerful, robust, versatile
- cost : 1-10 k€

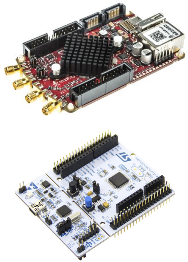


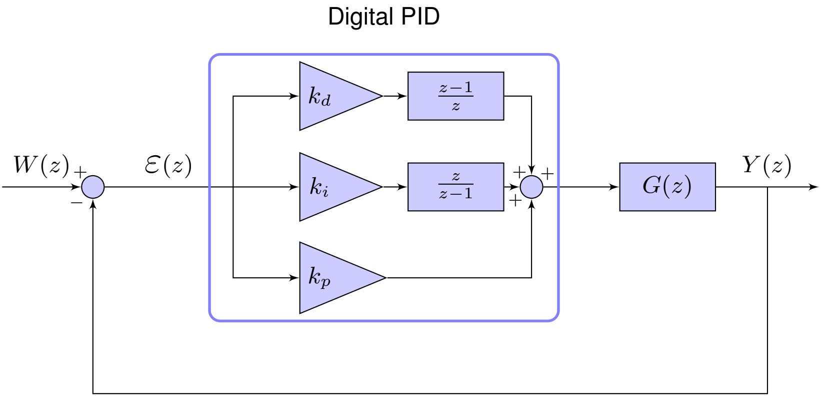
Figure: Red Pitaya - STM32.

- cost : 10-500 €

Warning !!!!

- F_s=1 Ms/s means
  - F_N=500 kHz
  - But just with a basic PID $F_{CL} \simeq 50$ kHz
- cost of the anti-aliasing filters $>>$ μcontroller !!

Digital PID



Improvements

1. Filtered PID :

$$\frac{U(z)}{\mathcal{E}(z)} = k_p + k_i T_s \frac{z}{z-1} + \frac{k_d}{T_s}\frac{z-1}{z-\alpha} \qquad \alpha \simeq 0.1$$

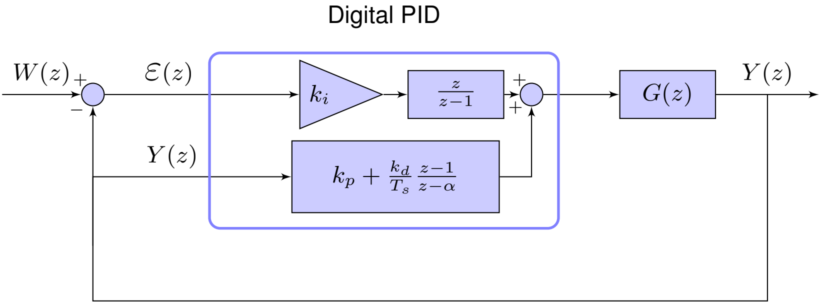
2. PID with derivative on the output :
   - tracking $\neq$ rejection

$$U(z) = k_p \mathcal{E}(z) + k_i T_s \frac{z}{z-1}\mathcal{E}(z) - \frac{k_d}{T_s}\frac{z-1}{z-\alpha}Y(z)$$

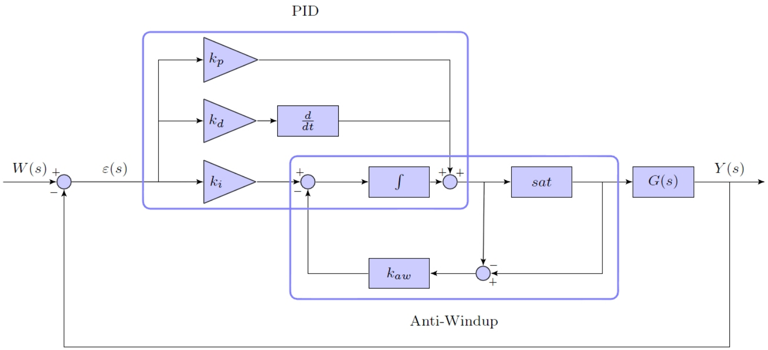
3. PID with proportional and derivative action on the output :
   - tracking $\neq$ rejection

$$U(z) = k_i T_s \frac{z}{z-1}\mathcal{E}(z) - \left[k_p + \frac{k_d}{T_s}\frac{z-1}{z-\alpha}\right]Y(z)$$

Digital PID


Digital PID
$W(z)$
+
−
$\mathcal{E}(z)$
k_i
$\frac{z}{z-1}$
+
+
$G(z)$
$Y(z)$
$Y(z)$
$k_p + \frac{k_d}{T_s}\frac{z-1}{z-\alpha}$

Anti windup

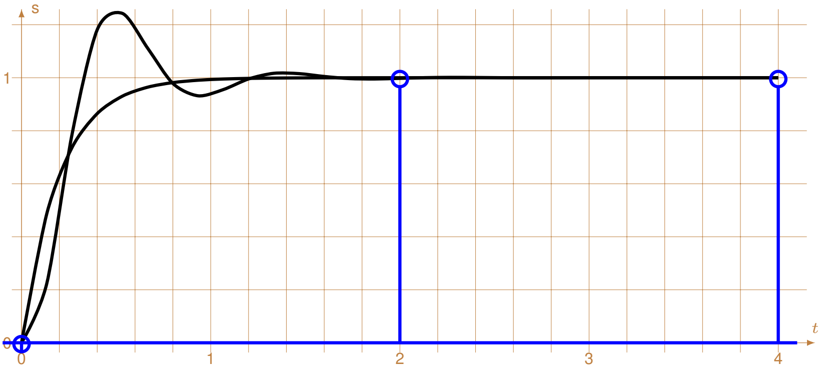

PID
k_p
k_d
$\frac{d}{dt}$
$W(s)$
$\varepsilon(s)$
k_i
$\int$
sat
$G(s)$
$Y(s)$
k_{aw}
Anti-Windup

PID code

```
int uk=PID(float ck, int yk, int ukmax, int ukmin, int uN){  /
// definition des variables locales                    // de varier
static int ukm1, uksm1, ykm1, ykm2;                    // variable
long int ukt; // uk temporaire
// calcul de l'erreur
ek=(float)(ck-yk);
// calcul de la commande
ukt=ukm1+(int)(((ki + k_{aw}*(uksm1-ukm1))*ek + (kp+ ki + kd/Te
uksm1=max(min(ukt,ukmax),ukmin); // saturation... peut être bie
uk=max(min(uksm1,32768-uN),-32768+uN); // j'aime bien garder ce
                                       // surtout en phase de 
// Mise à jour des variables
ukm1 =uk;
ykm1 =yk;
ykm2 =ykm1;
return uk+uN;
}
```

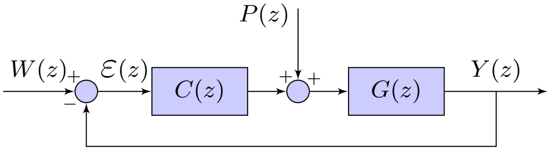
Dead-Bit controller

$$G(p) = ??? \text{ undersampling} \rightarrow G(z) = \frac{k(1-a)z^{-1}}{1-az^{-1}} \text{ or } G(z) = kz^{-1}$$

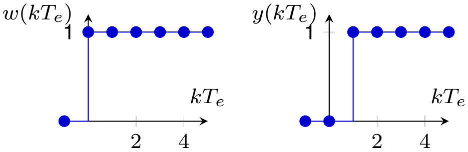


- Minimum settling time

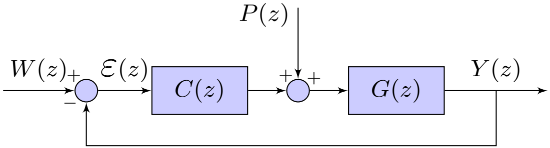
Dead-Bit controller - minimum settling time



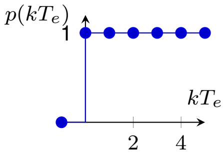
$$C(z) = \frac{1 - az^{-1}}{k(1-a)(1-z^{-1})} = \frac{k_p + k_i T_e - k_p z^{-1}}{1 - z^{-1}}$$

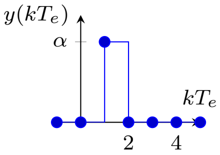


Dead-Bit controller - minimum rejection time



$$C(z) = \frac{\frac{a+1}{k(1-a)} - \frac{a}{k(1-a)} z^{-1}}{1 - z^{-1}}$$





State space representation

In a most general case :

$$\begin{aligned} \dot{\underline{x}} &= \mathbf{A}\underline{x} + \mathbf{B}\underline{u} + \underline{w} \\ \underline{y} &= \mathbf{C}\underline{x} + \mathbf{D}\underline{u} + \underline{v} \end{aligned}$$

w : Process noise
v : Output noise

- Can handle compactly multi variable systems.
- Poles of the transfer function $\Longleftrightarrow$ eigenvalues of $\mathbf{A}$

Linear quadratic control

Objective : minimize a quadratic criteria :

$$J = \int_0^\infty (\underline{x}^T \mathbf{Q} \underline{x} + \underline{u}^T \mathbf{R} \underline{u}) dt \tag{1}$$

with $\mathbf{Q}$ and $\mathbf{R}$ are positive semi-definite and positive definite :
The input u is then defined by:

$$u = -\mathbf{R}^{-1} \mathbf{B}^T \mathbf{K} \underline{x}$$

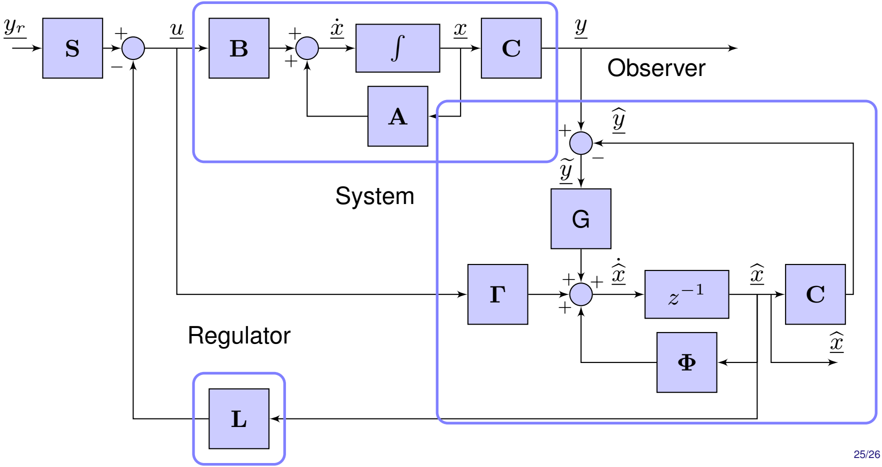
where $\mathbf{K}$ is a matrix, defined negative, solution of the Riccati's equation :

$$\mathbf{K}\mathbf{A} + \mathbf{A}^T \mathbf{K} - \mathbf{K}\mathbf{B}\mathbf{R}^{-1}\mathbf{B}^T\mathbf{K} + \mathbf{Q} = 0$$

Linear quadratic control

- Choice of $\mathbf{Q}$ and $\mathbf{R}$
  - Real energetic criteria
  - Minimize saturations
  - Any choice gives quite good behaviour
- Easy to calculate and implement (Matlab !)
- Allways stable (margin phase > 60°)

Digital Closed loop with observer



Kalman

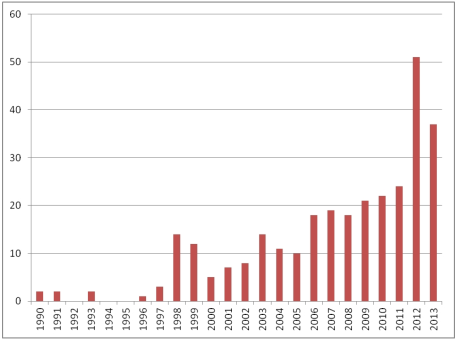


Figure: Citations of Kalman in EFTF

Conclusion

To go further,

- Perturbation estimation, computed torque control
- Luenberger & Kalman estimators
- Fuzzy logic control, Neural Networks
- Robust control (H_∞)
- Predictive control
- System identification
- ....

Utra short bibliography

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